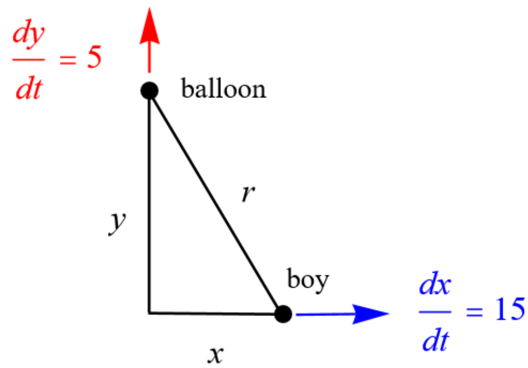
Exercise 99

A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

Solution

Draw a schematic of the boy and the balloon at a certain time after he passes under the balloon.



The aim is to find dr/dt when $t = 3$. The Pythagorean theorem relates the sides of a right triangle.

$$r^2 = x^2 + y^2$$

Differentiate both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2r \cdot \frac{dr}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

Divide both sides by 2.

$$r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

Divide both sides by r.

$$\frac{dr}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{r}$$

$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

$$= \frac{x(15) + y(5)}{\sqrt{x^2 + y^2}}$$

At $t = 3$, the distance the boy has travelled is $x = x_0 + v_x t = 0 + 15(3) = 45$ feet, and the distance the balloon has travelled is $y = y_0 + v_y t = 45 + 5(3) = 60$ feet. Therefore, the rate that the distance between the boy and the balloon is increasing when $t = 3$ s is

$$\left.\frac{dr}{dt}\right|_{t=3} = \frac{(45)(15) + (60)(5)}{\sqrt{(45)^2 + (60)^2}} = 13\ \frac{\text{ft}}{\text{s}}.$$